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INTEGRALS INVOLVING THE PRODUCT OF THE MULTIVARIABLE I-FUNCTION, GENERAL SEQUENCE OF FUNCTIONS AND A GENERAL CLASS OF POLYNOMIALS

V. G. GUPTA[1] & NAWAL KISHOR JANGID[2]

[1]Department of Mathematics, University of Rajasthan, Jaipur, Rajasthan, India

[2]Department of Mathematics, Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur, Rajasthan, India

ABSTRACT

In the present paper, we have established four integrals involving the product of multivariable I-function, general sequence of functions and a general class of polynomials. On account of general nature of the results established here, a number of known and new results follow as their particular cases on suitable specifications of the parameters involved there in. To illustrate, we have recorded some particular cases of our main results.

KEYWORDS: Multivariable I-Function, General Sequence of Functions and General Class of Polynomials

AMS Classification: 33C45, 33C60, 26D20

1. INTRODUCTION

We recall here the definition of I-function of 'r' variables given by Prithma Jayarama et al. [4] in the following manner

$$I\left[z_{1,\dots,}z_r\right] =$$

$$I^{\,0,\,n:\,(m_1,n_1);\dots;(m_r,n_r)}_{\,p,q:(p_1,q_1);\dots;(p_r,q_r)}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} (a_j;\alpha_j^{(1)},\dots,\alpha_j^{(r)};A_j)_{1,p}: (c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r} \\ (b_j;\beta_j^{(1)},\dots,\beta_j^{(r)};B_j)_{1,q}: (d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r} \end{matrix}\right]$$

$$= (2\pi\omega)^{-r}\int_{L_1}\dots\int_{L_r}\phi(\xi_1,\dots,\xi_r)\,\theta_1(\xi_1)\dots\theta(\xi_r)\,z_1^{\xi_1}\dots z_r^{\xi_r}\,d\xi_1\dots d\xi_r \tag{1.1}$$

Where

$$\phi(\xi_1,\dots,\xi_r) = \frac{\prod_{j=1}^{n}\Gamma^{A_j}[1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i]}{\prod_{j=n+1}^{p}\Gamma^{A_j}[a_j-\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i]\prod_{j=1}^{q}\Gamma^{B_j}[1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}\xi_i]} \tag{1.2}$$

And

$$\theta_i(\xi_i) = \frac{\prod_{j=1}^{n_i} \Gamma^{C_j^{(i)}}[1-c_j^{(i)} + \gamma_j^{(i)}\xi_i] \prod_{j=1}^{m_i} \Gamma^{D_j^{(i)}}[d_j^{(i)} - \delta_j^{(i)}\xi_i]}{\prod_{j=n_i+1}^{p_i} \Gamma^{C_j^{(i)}}[c_j^{(i)} - \gamma_j^{(i)}\xi_i] \prod_{j=m_i+1}^{q_i} \Gamma^{D_j^{(i)}}[1-d_j^{(i)} + \delta_j^{(i)}\xi_i]} \quad ; \tag{1.3}$$

Where i = 1,…,r., $z_i \neq 0$, and $\omega = \sqrt{-1}$. For more details and convergence conditions, one may refer to [4]

The series formula for the general sequence of functions due to Salim [5] is

$$R_n^{\alpha,\beta}[x ; A,B,c,d ; p,q ; \gamma,\delta;\vartheta,k;e^{-sx^{r'}}]$$

$$= \frac{B^{\gamma n} x^{kn} (cx^q + d)^{\delta n} k^n e^{sx^{r'}}}{k_n} \sum_{m,v,u,t,e} \frac{(-1)^{h+m}(-v)_u (-h)_e (\alpha)_h s^m}{m!\, v!\, u!\, t!\, e!}$$

$$\times \frac{(-\alpha-\gamma n)_e (-\beta-\delta n)_v}{(1-\alpha-h)_e} \left(\frac{pe + r'm + \vartheta + qu}{k}\right)_n \times \left(\frac{cx^q}{cx^q + d}\right)^v \left(\frac{Ax^p}{B}\right)^h x^{r'm}, \tag{1.4}$$

Where

$$\sum_{m,v,u,h,e} \equiv \sum_{m=0}^{\infty} \sum_{v=0}^{n} \sum_{u=0}^{v} \sum_{h=0}^{n} \sum_{e=0}^{h} \tag{1.5}$$

For the present study, we use the following special case of (1.4), obtained by setting s = 0 and expanding $\left(cx^q + d\right)^{-v+\delta n}$ in series

$$R_n^{\alpha,\beta}[x ; A,B,c,d ; p,q ; \gamma,\delta;\vartheta,k;1]$$

$$= \sum_{m,v,u,h,e} \psi(m,v,u,h,e) x^{kn+q(m+e)+pu} \tag{1.6}$$

Where

$$\psi(m,v,u,h,e) = \frac{A^u B^{\gamma n-u} k^n c^{m+e} d^{-m+n\delta-e} (-m)_v (-u)_h (\alpha)_u}{k_n\, m!\, v!\, u!\, h!\, e!}$$

$$\times \frac{(-1)^{u+e} (-\alpha-\gamma n)_h (-\beta-\delta n)_m (m-\delta n)_e}{(1-\alpha-u)_h} \left(\frac{ph + \vartheta + qv}{k}\right)_n \tag{1.7}$$

The general class of polynomials, occurring in this paper was introduced by Srivastava [6] and defined as

$$S_N^M[x] = \sum_{l=0}^{[N/M]} \frac{(-N)_{Ml}}{l!} A_{N,l}\, x^l, \; N = 0,1,2,... \tag{1.8}$$

Where M is an arbitrary positive integer and the coefficients $A_{N,l}(N,l > 0)$ are arbitrary constants, real or complex.

2. THE MAIN INTEGRAL FORMULAE

The following integral formulae have been established in this section:

First Integral

$$\int_0^\infty t^{\alpha'-1/2}(t+a)^{-\alpha'}(t+b)^{-\alpha'} S_N^M\left[\left(\frac{t}{(t+a)(t+b)}\right)^{\sigma}\right]\times$$

$$R_n^{(\alpha,\beta)}\left[\left(\frac{t}{(t+a)(t+b)}\right)^{\mu};A,B,c,d;p,q;\gamma,\delta;\vartheta,k;1\right]\times$$

$$I\begin{matrix}0,\,n:\,(m_1,n_1);...;(m_r,n_r)\\ p,q:(p_1,q_1);...;(p_r,q_r)\end{matrix}\left[\left\{\frac{t}{(t+a)(t+b)}\right\}^{\rho_1} z_1,...,\left\{\frac{t}{(t+a)(t+b)}\right\}^{\rho_r} z_r\right]dt$$

$$=\sum_{l=0}^{[N/M]}\frac{(-N)_{Ml}}{l!}A_{N,l}\sum_{m,v,u,h,e}\psi(m,v,u,h,e)\sqrt{\pi}(\sqrt{a}+\sqrt{b})^{1-2(\alpha'+\sigma l+\mu w)}$$

$$I\begin{matrix}0,\,n+1:\,(m_1,n_1);...;(m_r,n_r)\\ p+1,q+1:(p_1,q_1);...;(p_r,q_r)\end{matrix}\left[\begin{matrix}z_1(\sqrt{a}+\sqrt{b})^{-2\rho_1}\\ \vdots\\ z_r(\sqrt{a}+\sqrt{b})^{-2\rho_r}\end{matrix}\middle|\begin{matrix}(3/2-\alpha-\sigma l-\mu w;\rho_1,...,\rho_r;1),\\ (1-\alpha-\sigma l-\mu w;\rho_1,...,\rho_r;1),\end{matrix}\right.$$

$$\left.\begin{matrix}(a_j;\alpha_j^{(1)},...,\alpha_j^{(r)};A_j)_{1,p}:(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};...;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},...,\beta_j^{(r)};B_j)_{1,q}:(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};...;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r}\end{matrix}\right] \tag{2.1}$$

Provided that $a,b,\sigma,\mu,\rho_i(i=1,...,r)$ are all positive, $\mathrm{Re}(\alpha')>0$

And

$$\mathrm{Re}\left(\alpha'+\frac{1}{2}\right)+\sum_{i=1}^{r}\rho_i\min_{1\le j\le\mu^{(i)}}\left[\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0 \tag{2.2}$$

Where

$$w=kn+q(m+e)+pu,$$

Second Integral

$$\int_0^t x^{-\alpha'-1}(t-x)^{\alpha'-1}e^{-\beta'/x}S_N^M\left[\left(\frac{t-x}{x}\right)^{\sigma}\right]\times R_n^{(\alpha,\beta)}\left[\left(\frac{t-x}{x}\right)^{\mu};A,B,c,d;p,q;\gamma,\delta;\vartheta,k;1\right]\times$$

$$I^{0,\, n\,:\,(m_1,n_1)\,;...;\,(m_r,n_r)}_{p,q\,:(p_1,q_1)\,;...;\,(p_r,q_r)}\left[\left\{\frac{t-x}{x}\right\}^{\rho_1} z_1,...,\left\{\frac{t-x}{x}\right\}^{\rho_r} z_r\right]dx$$

$$=\sum_{l=0}^{[N/M]}\frac{(-N)_{Ml}}{l!}A_{N,l}\sum_{m,v,u,h,e}\psi(m,v,u,h,e)\beta'^{-\alpha'-\sigma l-\mu w}\, t^{\alpha'+\sigma l+\mu w-1}e^{-\beta'/t}$$

$$I^{0,\, n+1\,:\,(m_1,n_1)\,;...;\,(m_r,n_r)}_{p+1,q\,:(p_1,q_1)\,;...;\,(p_r,q_r)}\left[\begin{array}{c} z_1(t/\beta')^{\rho_1}\\ \vdots \\ z_r(t/\beta')^{\rho_r}\end{array}\left|\begin{array}{c}(1-\alpha'-\sigma l-\mu w;\rho_1,...,\rho_r;1),\\ ------\end{array}\right.\right.$$

$$\left.\begin{array}{l}(a_j\,;\alpha_j^{(1)},...,\alpha_j^{(r)};A_j)_{1,p}:(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};...;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r}\\ (b_j\,;\beta_j^{(1)},...,\beta_j^{(r)};B_j)_{1,q}:(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};...;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r}\end{array}\right] \tag{2.3}$$

Provided that $\sigma,\mu,\rho_i(i=1,...,r)$ are all positive

$\mathrm{Re}(\alpha')>0$, $\mathrm{Re}(\beta')>0$ (2.4)

And

$$\mathrm{Re}(\alpha')+\sum_{i=1}^{r}\rho_i\min_{1\le j\le\mu^{(i)}}\left[\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0 \tag{2.5}$$

Third Integral

$$\int_0^1\!\!\int_0^1\left(\frac{1-x}{1-xy}y\right)^{\alpha'}\left(\frac{1-y}{1-xy}\right)^{\beta'}\left[\frac{1-xy}{(1-x)(1-y)}\right]S_N^M\left[\left(\frac{1-y}{1-xy}\right)^{\sigma}\right]\times$$

$$R_n^{(\alpha,\beta)}\left[\left(\frac{1-y}{1-xy}\right)^{\mu}:A,B,c,d;p,q:\gamma,\delta;\vartheta,k;1\right]\times$$

$$I^{0,\, n\,:\,(m_1,n_1)\,;...;\,(m_r,n_r)}_{p,q\,:(p_1,q_1)\,;...;\,(p_r,q_r)}\left[\left\{\frac{1-x}{1-xy}y\right\}^{\rho_1}\left\{\frac{1-y}{1-xy}\right\}^{\tau_1} z_1,...,\left\{\frac{1-x}{1-xy}y\right\}^{\rho_r}\left\{\frac{1-y}{1-xy}\right\}^{\tau_r} z_r\right]dxdy$$

$$=\sum_{l=0}^{[N/M]}\frac{(-N)_{Ml}}{l!}A_{N,l}\sum_{m,v,u,h,e}\psi(m,v,u,h,e)$$

$$I^{0,\, n+2\,:\,(m_1,n_1)\,;...;\,(m_r,n_r)}_{p+2,q+1:(p_1,q_1)\,;...;\,(p_r,q_r)}\left[\begin{array}{c} z_1\\ \vdots \\ z_r\end{array}\left|\begin{array}{c}(1-\alpha',\rho_1,...,\rho_r;1),(1-\beta'-\sigma l-\mu w,\tau_1,...,\tau_r;1),\\ (1-\alpha'-\beta'-\sigma l-\mu w,\sum_{i=1}^{r}(\rho_i+\tau_i);1),\end{array}\right.\right.$$

$$\left.\begin{matrix}(a_j\ ;\alpha_j^{(1)},...,\alpha_j^{(r)};A_j)_{1,p}:(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};...;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r}\\(b_j\ ;\beta_j^{(1)},...,\beta_j^{(r)};B_j)_{1,q}:(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};...;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r},\end{matrix}\right] \quad (2.6)$$

Where $\rho_i,\sigma,\tau_i>0$,

$$\mathrm{Re}(\alpha')+\sum_{i=1}^{r}\rho_i\min_{1\le j\le\mu^{(i)}}\left[\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0$$

$$\mathrm{Re}(\beta')+\sum_{i=1}^{r}\rho_i\min_{1\le j\le\mu^{(i)}}\left[\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0 \quad (2.7)$$

Fourth Integral

$$\int_0^1\int_0^1 f(x,y)y^{\alpha'}(1-x)^{1-\alpha'}(1-y)^{\beta'-1}S_N^M[(1-y)^{\sigma}]\times R_n^{(\alpha,\beta)}\left[(1-y)^{\mu}:A,B,c,d;p,q:\gamma,\delta;\vartheta,k;1\right]\times$$

$$I_{p,q:(p_1,q_1);...;(p_r,q_r)}^{0,n:(m_1,n_1);...;(m_r,n_r)}\left[z_1y^{\rho_1}(1-y)^{\tau_1}(1-x)^{\rho_1},...,z_ry^{\rho_r}(1-y)^{\tau_r}(1-x)^{\rho_r}\right]dxdy$$

$$=\sum_{l=0}^{[N/M]}\frac{(-N)_{Ml}}{l!}A_{N,l}\sum_{m,v,u,h,e}\psi(m,v,u,h,e)\int_0^1 f(z)(1-z)^{\alpha'+\beta'+\sigma l+\mu w+1}\times$$

$$I_{p+2,q+1:(p_1,q_1);...;(p_r,q_r)}^{0,n+2:(m_1,n_1);...;(m_r,n_r)}\left[\begin{matrix}z_1(1-z)^{\rho_1+\sigma_1}\\ \vdots\\ z_r(1-z)^{\rho_r+\sigma_r}\end{matrix}\left|\begin{matrix}(1-\alpha',\rho_1,...,\rho_r;1),(1-\beta'-\sigma l-\mu w,\tau_1,...,\tau_r;1),\\(1-\alpha'-\beta'-\sigma l-\mu w,\sum_{i=1}^{r}(\rho_i+\tau_i);1),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j\ ;\alpha_j^{(1)},...,\alpha_j^{(r)};A_j)_{1,p}:(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};...;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r}\\(b_j\ ;\beta_j^{(1)},...,\beta_j^{(r)};B_j)_{1,q}:(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};...;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r}\end{matrix}\right]dz \quad (2.8)$$

Where $\rho,\sigma,\tau>0$,

$$\mathrm{Re}(\alpha')+\sum_{i=1}^{r}\rho_i\min_{1\le j\le\mu^{(i)}}\left[\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0$$

$$\mathrm{Re}(\beta')+\sum_{i=1}^{r}\rho_i\min_{1\le j\le\mu^{(i)}}\left[\mathrm{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right]>0 \quad (2.9)$$

Proofs: To establish (2.1), we first use the expressions (1.1) and (1.8) in the L.H.S. of (2.1) and get L.H.S. of (2.1)

$$\left[\int_0^\infty t^{\alpha'-1/2}(t+a)^{-\alpha'}(t+b)^{-\alpha'}\sum_{l=0}^{[N/M]}\frac{(-N)_{Ml}}{l!}A_{M,l}\left[\left(\frac{t}{(t+a)(t+b)}\right)^{\sigma l}\right]\times\right.$$

$$\sum_{m,v,u,h,e}\psi(m,v,u,h,e)\frac{t^{\mu w}}{(t+a)^{\mu w}(t+b)^{\mu w}}\times$$

$$\frac{1}{(2\pi\omega)^{-r}}\int_{L_1}\ldots\int_{L_r}\phi(\xi_1,\ldots,\xi_r)\,\theta_1(\xi_1)\ldots\theta(\xi_r)\left\{\frac{t}{(t+a)(t+b)}\right\}^{\sum_{i=1}^{r}\rho_i\xi_i} z_1^{\xi_1}\ldots z_r^{\xi_r} \tag{2.10}$$

$$\left.d\xi_1,\ldots,d\xi_r\right]dt$$

Where $\phi(\xi_1,\ldots,\xi_r)$ and $\theta_1(\xi_1),\ldots,\theta_r(\xi_r)$ are given by (1.2) and (1.3) respectively.

Further on changing the order of integration and summation in (2.10) under the given condition and then arranging the terms involving powers of t, (t+a), (t+b) etc. , we arrive at

L.H.S. of (2.1)

$$\sum_{l=0}^{[N/M]}\frac{(-N)_{Ml}}{l\,!}A_{N,l}\sum_{m,v,u,h,e}\psi(m,v,u,h,e)$$

$$\frac{1}{(2\pi\omega)^{-r}}\left\{\int_{L_1}\ldots\int_{L_r}\phi(\xi_1,\ldots,\xi_r)\,\theta_1(\xi_1)\ldots\theta(\xi_r)\,z_1^{\xi_1}\ldots z_r^{\xi_r}\right.$$

$$\left.\left(\int_0^\infty t^{\alpha'+\mu w+\sigma l+\sum_{i=1}^{r}\rho_i\xi_i-1/2}(t+a)^{-\alpha'-\mu w-\sigma l-\sum_{i=1}^{r}\rho_i\xi_i}(t+b)^{-\alpha'-\mu w-\sigma l-\sum_{i=1}^{r}\rho_i\xi_i}dt\right)d\xi_1,\ldots,d\xi_r\right\} \tag{2.11}$$

Now, evaluating the inner integral in (2.11) by using the following result due to Gradshteyn and Ryzhik [2, p. 317, 3.197(7)]

$$\int_0^\infty x^{\alpha'-1/2}(x+a)^{-\alpha'}(x+b)^{-\alpha'}dx=\sqrt{\pi}(\sqrt{a}+\sqrt{b})^{1-2\alpha'}\frac{\Gamma\alpha'-\frac{1}{2}}{\Gamma\alpha'},\quad \mathrm{Re}(\alpha')>0 \tag{2.12}$$

We get the required result (2.1).

The results (2.3),(2.6) and (2.8) can be established on the lines similar to those mentioned for (2.1),by using the following results [2, p.339, eq. 3.471(3); [1], p.145, 243)]

$$\int_0^t x^{-\alpha-1}(t-x)^{\alpha-1}e^{-\beta/x}dx=\Gamma(\alpha)\,\beta^{-\alpha}t^{\alpha-1}e^{-\beta/t} \tag{2.13}$$

$$\int_0^1\int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{-\alpha-\beta+1}dxdy=B(\alpha,\beta)=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \tag{2.14}$$

$$\int_0^1\int_0^1 f(x,y)y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}dxdy=B(\alpha,\beta)\int_0^1 f(t)(1-t)^{\alpha+\beta+1}dt \tag{2.15}$$

3. SPECIAL CASES

- **On Setting**

$$\mathrm{M}\ =\ 1,\ \mathrm{A}_{\mathrm{N,s}}=\binom{N+\gamma}{N}\frac{(\gamma+\beta+N+1)_{\mathrm{s}}}{(\gamma+1)_{\mathrm{s}}}\ ,$$

And then on using the result [7, pp. 68, Eq. (4.3.2)]

$$\mathrm{S}_N^1\,[\mathrm{x}]\ =\ \mathrm{P}_N^{(\gamma,\beta)}\,[1-2\mathrm{x}],$$

Where $\mathrm{P}_{\mathrm{n}}^{(\gamma,\beta)}[x]$, denotes the Jacobi polynomials in (2.1), we get the following interesting integral formula in terms of Jacobi polynomials

$$\int_0^{\infty} t^{\alpha'-1/2}(t+a)^{-\alpha'}(t+b)^{-\alpha'}P_N^{(\gamma,\beta)}\left[1-2\left(\frac{t}{(t+a)(t+b)}\right)^{\sigma}\right]\times$$

$$R_N^{(\alpha,\beta)}\left[\left(\frac{t}{(t+a)(t+b)}\right)^{\mu};A,B,c,d;p,q;\gamma,\delta;\vartheta,k;1\right]\times$$

$$I^{\,0,\,n\,:\,(m_1,n_1)\,;...;\,(m_r,n_r)}_{\,p,q\,:(p_1,q_1)\,;...;\,(p_r,q_r)}\left[\left\{\frac{t}{(t+a)(t+b)}\right\}^{\rho_1}z_1,...,\left\{\frac{t}{(t+a)(t+b)}\right\}^{\rho_r}z_r\right]dt$$

$$=\sum_{l=0}^{N}\frac{(1+\alpha')_N(1+\alpha'+\beta'+N)_l}{l!(N-l)!(1+\alpha')_{k'}}\sum_{m,v,u,h,e}\psi(m,v,u,h,e)\sqrt{\pi}(\sqrt{a}+\sqrt{b})^{1-2(\alpha'+\sigma l+\mu w)}$$

$$I^{\,0,\,n+1\,:\,(m_1,n_1)\,;...;\,(m_r,n_r)}_{\,p+1,q+1\,:(p_1,q_1)\,;...;\,(p_r,q_r)}\left[\begin{matrix} z_1(\sqrt{a}+\sqrt{b})^{-2\rho_1}\\ \vdots \\ z_r(\sqrt{a}+\sqrt{b})^{-2\rho_r}\end{matrix}\left|\begin{matrix}(3/2-\alpha'-\sigma l-\mu w;\rho_1,...,\rho_r;1),\\ (1-\alpha'-\sigma l-\mu w;\rho_1,...,\rho_r;1),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j\,;\alpha_j^{(1)},...,\alpha_j^{(r)};A_j)_{1,p}:(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};...;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r}\\ (b_j\,;\beta_j^{(1)},...,\beta_j^{(r)};B_j)_{1,q}:(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};...;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r}\end{matrix}\right] \tag{3.1}$$

Under the above specification of parameters, the results (2.3), (2.6) and (2.8) can also be obtained in terms of Jacobi polynomials.

(iii) If we put $p=d=1, K_n=1, s=0$ and take $n=q=\lambda=B=0, l=r=-1$ and $A=1$ in (2.1) then $R_n^{(\alpha,\beta)}(x)$ reduces to unity and we obtain

$$\int_0^\infty t^{\alpha'-1/2}(t+a)^{-\alpha'}(t+b)^{-\alpha'} S_N^M\left[\left(\frac{t}{(t+a)(t+b)}\right)^\sigma\right]\times$$

$$I\,{}^{0,\,n\,:\,(m_1,n_1)\,;...;\,(m_r,n_r)}_{p,q\,:(p_1,q_1)\,;...;\,(p_r,q_r)}\left[\left\{\frac{t}{(t+a)(t+b)}\right\}^{\rho_1} z_1,...,\left\{\frac{t}{(t+a)(t+b)}\right\}^{\rho_r} z_r\right]dt$$

$$=\sum_{l=0}^{[N/M]}\frac{(-N)_{Ml}}{l!}A_{N,l}\sqrt{\pi}(\sqrt{a}+\sqrt{b})^{1-2(\alpha'+\sigma l)}$$

$$I\,{}^{0,\,n+1:\,(m_1,n_1)\,;...;\,(m_r,n_r)}_{p+1,q+1:(p_1,q_1)\,;...;\,(p_r,q_r)}\left[\begin{matrix} z_1(\sqrt{a}+\sqrt{b})^{-2\rho_1}\\ \vdots \\ z_r(\sqrt{a}+\sqrt{b})^{-2\rho_r}\end{matrix}\left|\begin{matrix}(3/2-\alpha'-\sigma l;\rho_1,...,\rho_r;1),\\ (1-\alpha'-\sigma l;\rho_1,...,\rho_r;1),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j\,;\alpha_j^{(1)},...,\alpha_j^{(r)};A_j)_{1,p}:(c_j^{(1)},\gamma_j^{(1)};C_j^{(1)})_{1,p_1};...;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1,p_r}\\ (b_j\,;\beta_j^{(1)},...,\beta_j^{(r)};B_j)_{1,q}:(d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};...;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r}\end{matrix}\right] \quad (3.2)$$

(iii)On putting $A_j(j=1,...,p), B_j(j=1,...,q), C_j^{(i)}(j=1,...,p_i, i=1,...,r)$ and $D_l^{(i)}(j=1,...,q_i, i=1,...,r)$ in (3.2) the I-function reduces to H-function of several complex variables and we get the results obtained by Gupta [3].

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